Introduction

The Sphaerica

Theodosius is known as the author of the Sphaerica, the De habitationibus and the De diebus et noctibus. Little is known of his life, but from citations by Strabo (d. ca. 25 AD)¹ he seems to have lived ca. 100 BC. His three works all belonged to the "little astronomy" – "little" in contrast to the great Astronomy of Claudius Ptolemy (2c. AD), later known as the Almagest. Arabic translations were considered to belong to the similar collection of mathematical and astronomical works called in the Arabic literature the "middle books" ("middle", that is, between the Elements of Euclid and Ptolemy's Almagest).

Unlike most of the "little astronomy", including Theodosius' *Opera minora*, the *Sphaerica* is presented as a purely geometric treatise, though it is clear that many of the propositions have an astronomical application². There are occasional lapses into astronomical language, e.g. "the visible pole" in some propositions of Books II and III. The style of the *Sphaerica* is that of Euclid's *Elements*: for each proposition there is an enunciation, a display of this by reference to a lettered diagram, and a proof. There are, too, correspondences of some of the early theorems (on spheres) and the third book of the *Elements* (on circles).

There are numerous versions and redactions of the *Sphaerica* in Arabic, Latin and Hebrew.

The Arabic Tradition

In Arabic there are two known translations, represented by **ANH** (edited here) and **FC**, and several redactions, those in **L**, **K**, the wellknown $Tahr\bar{i}r$ by Naṣīr al-Dīn al-Ṭūsī (d. 1274) and the redactions by Ibn Abī [al-]Shukr (d. between 1281 and 1291) and Taqī al-Dīn ibn Maʿrūf (d. 1585)³.

¹On Theodosius' life and works see R. Fecht, "De Theodosii vita et scriptis", in his edition of *De habitationibus* and *De diebus et noctibus*; also Czinczenheim, 8–17, Ziegler, Bulmer-Thomas, the introduction to Ver Eecke's French translation of the *Sphaerica*, and Heath [1921], II, 245-246.

²See Czinczenheim, 957-984. For a description of the mathematical content of the work, see also Heath [1921], II, 246–252, and Ver Eecke, *op. cit.*, xii–xix.

³For Taqī al-Dīn, see Ḥājjī Khalīfa, col. 142.

In the colophon of \mathbf{L} it is said that the manuscript was copied from a copy which was copied from a copy in the hand of Ibn al-Sarī (i.e. the famous commentator of Arabic translations of Greek mathematical and philosophical works, also known as Ibn al-Salāḥ, d. 1154).

The Arabic title of the *Sphaerica* was not literally translated from the Greek title, $\Sigma \varphi \alpha i \varphi i \alpha \lambda \alpha i \alpha$. It appears in various forms: *Kitāb al* $ukar^4$, *Kitāb al-kurāt*⁵ and *Kitāb al-kura*⁶ all occur, the first being the most frequent.

No translator is mentioned in the bio-bibliographical literature: the *Fihrist* of Ibn al-Nadīm and Ibn al-Qifţī's biographical work. They mention Theodosius' three works, but give no translator for any of them⁷. Of **ANH**, only **N** mentions that the text was revised by Thābit ibn Qurra (*işlāḥ*); but at the beginning of Book II it is said, instead, that it is a translation (*tarjama*) by Thābit. In **FC** it is stated that Qusṭā b. Lūqā translated it (*tarjama*). At the beginning of the text in **L** it is said that it was the translation (*naql*) of Abū Zayd Ḥunayn b. Isḥāq. However, this ascription is unlikely, because Ḥunayn is mostly known for his translations of medical and philosophical works. At the beginning of **K**, which is similar to Ṭūsī (see below), the scribe writes that Aḥmad, son of Caliph al-Muʿtaṣim⁸, ordered the *Sphaerica* to be brought from Greek into Arabic (*ikhrāj*) and that Qusṭā b. Lūqā translated (*tarjama*) it.

Most detailed is the description in the introduction to $T\bar{u}s\bar{r}$'s $Tahr\bar{r}r$ of the Sphaerica (completed in 1253), where he says: "There has ordered its translation (naql) from Greek into Arabic Abū 'l-'Abbās Ahmad ibn al-Mu'taṣim bi-llāh, and Qusṭā ibn Lūqā al-Ba'labakkī undertook its translation (naql) until the fifth proposition of the third book; then somebody else (ghayruhu) undertook the translation of the rest, and Thābit ibn Qurra revised it (aslahahu)"⁹.

 $T\bar{u}s\bar{i}s$ text was later repeated almost *verbatim* by $H\bar{a}jj\bar{i}$ Khalīfa (d. 1657) in his description of the *Sphaerica*, *Kitāb al-ukar*: "There has ordered its translation (*naql*) from Greek into Arabic al-Musta'īn bi-

⁴Thus in \mathbf{A} , \mathbf{K} and $\overline{T}\overline{u}s\overline{i}$.

 $^{{}^{5}\}mathbf{N}$ (at the end of Book III).

 $^{^{6}\}mathbf{H}$ (at the beginning) and **FC**.

 $^{^7\}mathrm{Ibn}$ al-Nadīm, 269, lines 5–7, the name being wrongly given as Thywdwrs; Ibn al-Qift_ī, 108, lines 1–5 and 11–14.

 $^{^8}Sic;$ in reality Aḥmad was the grandson of al-Muʿtaṣim and later became caliph himself with the honorary title of al-Mustaʿīn (r. 862–866). Al-Muʿtaṣim reigned 833–842.

 $^{{}^{9}}$ Ţūsī, 2.

llāh Abū 'l-ʿAbbās Aḥmad ibn al-Muʿtaṣim during his caliphate, and Qusṭā ibn Lūqā al-Baʿlabakkī undertook its translation (*naql*) until the fifth proposition of the second (*sic*)¹⁰ around the year 250¹¹; then somebody else (*ghayruhu*) undertook the translation of the rest, and Thābit ibn Qurra revised it (*aṣlaḥahu*). Then the learned Naṣīr al-Dīn Muḥammad ibn Muḥammad al-Ṭūsī, who died in the year 672 (= 1274 AD), and the excellent Taqī al-Dīn Muḥammad ibn Maʿrūf al-Rāṣid, who died in the year 993 (= 1585 AD), made recensions of it"¹².

The Arabic Manuscripts

 A^{13} : Istanbul, Seray, Ahmet III 3464, ff. 20v–53v. This codex contains altogether seventeen treatises – *inter alia* most of the middle books – in several hands¹⁴. Seven of them, among them the *Sphaerica*, are in the same hand, three of which (but not the *Sphaerica*) are dated to August or September 1228 and written by Muḥammad b. Abī Bakr b. Muḥammad. Three other texts, in different hands, are dated to 1219, 1233 and 1290.

N: Lahore, private library M. Nabī Khān, pp. 185–281. This codex contains, besides the *Sphaerica*, also a copy of the *De habitationibus* (pp. 282–294)¹⁵. The first two pages of the *Sphaerica* (pp. 185, 186) are lost and have been supplied, in another hand, by two pages from $T\bar{u}s\bar{i}$'s *Taḥrīr*. The genuine text begins at the top of p. 187, with *almustaqīma* (Prop. I 1, line 5, in the present edition). In the colophon the scribe explains that the text he copied was in the hand of a direct descendant of Thābit ibn Qurra. From his further report it is clear that, at some stage of the transmission, the diagrams were corrected by al-Ḥasan ibn Saīd:

Finished is the third chapter of Theodosius' book on the spheres, and with its ending the entire book is finished with the praise of God. It is fourteen theorems and the number of the theorems of

 $^{{}^{10}}al$ -thāniya, the second, is an easy miswriting in Arabic script for al-thālitha, the third; the word for "Book" (al-maqāla) has been omitted here.

 $^{^{11}}$ I.e. 864 AD. This detail is not in <code>Ţūsī</code>. The given year falls in the reign of Caliph al-Mustaʿīn.

 $^{^{12}}$ Hājjī Khalīfa, col. 142.

 $^{^{13}\}mathrm{From}~\mathbf{A}$ the text has been edited in the unpublished dissertation by T. J. Martin, University of St. Andrews, 1975.

¹⁴For a detailed list of the items, see Lorch [2001], 22-23.

 $^{^{15}}$ We are very grateful to the late Dr. Anton Heinen for giving us copies of the Theodosius texts in this manuscript. Unfortunately, we have no access to the rest of this manuscript and cannot describe it.

the three chapters is 59, [in] the correction by Thābit b. Qurra al-Ḥarrānī al-Ṣābi'.

I have copied this book from the handwriting of Qurra b. Sīnān b. Manṣūr b. Saʿīd b. Thābit b. Sinān b. Thābit b. Qurra al-Ḥarrānī al-Ṣābi' in the city of Mosul (God protect it!) in the Niẓāmīya Madrasa (God give it long life!), when six nights remained of Jumādā I of the year 554 H [= 13 June 1158] (upon its patron be the finest *salām*!).

I found written at the end of the book: "al-Ḥasan b. Saʿīd has finished devising the diagrams $[tashk\bar{\imath}l]$ of this book, but the volume from which he copied the figures $[ashk\bar{\imath}l]$ was not reliable. Moreover there was corruption in it, so it was necessary to collate it with the figures $[ashk\bar{\imath}l]$ in another copy. That was on the eve of Tuesday, eight nights remaining of twelve [i.e. Dhū 'l-Ḥijja] of the year 421 [20 December 1030]. Praise be to God richly and His blessings upon Muhammad and all his family!"

Some notes on the text by al-Ḥasan ibn Saʿīd are edited below, after the text itself.

H: Paris, BnF hebr. 1101, ff. 1–53r, 86r–87r, in Hebrew script. This contains, besides the *Sphaerica*, only one other text¹⁶: the treatise on the use of the astrolabe by Abū [al-]Ṣalt Umayya b. 'Abd al-'Azīz b. Abī [al-]Ṣalt (d. 1034) in 88 chapters¹⁷. At the beginning this is entitled *Risāla (sic) al-asṭurlāb li-Abī Ṣalt*. At the end it is called *Kitāb al-'amal bi-l-asṯurlāb*.

These three manuscripts, \mathbf{ANH} , represent the translation that is edited here. Another translation is represented by \mathbf{FC} . Further, \mathbf{L} and \mathbf{K} appear to be two different reworkings of these translations. The details of these manuscripts are as follows:

F: Florence, Laur. Med. 124, 76ff., and **C**, Cambridge, University Library, Add. 1220, ff. 1r–50r, are both in Hebrew script. They seem to represent another translation, which both manuscripts attribute to Qustā ibn Lūqā.

L: Leiden, Or. 1031, pp. 22–72. According to the colophon this text seems to be a reworking by Ibn al-Ṣalāḥ (see above), despite the ques-

¹⁶Ff. 56r–85r, the intervening pages being blank and ff. 86r–87r containing Prop. III 7, which is omitted in the main copy.

 $^{^{17}\}mathrm{See}$ Steinschneider, 364.

tionable ascription of the translation in the manuscript to Hunayn b. Ishāq.

K: Private library (formerly in the possession of H. P. Kraus), ff. 33v– 64r, 7/13c., is also apparently a reworking. It is striking that the preface has a wording that is almost identical to $T\bar{u}s\bar{1}$'s preface in his *Tahrīr*. This seems to point to **K**'s being written after the *Tahrīr*. The author of **K** here used some terms different from $T\bar{u}s\bar{1}$'s and omitted the second part of $T\bar{u}s\bar{1}$'s preface on the details of Qustā's translation. Another possibility is that $T\bar{u}s\bar{1}$ and the author of **K** used the same source.

The Latin Tradition

Two Latin texts of the *Sphaerica* circulated in the Middle Ages, the one edited here and a longer version that has been ascribed to Campanus¹⁸. Our text is clearly a translation of the Arabic of **ANH**. Since it is of the translation style of Gerard of Cremona – literalism and some characteristic translations (e.g. *cum* for *idhā*, *si* for *in*, etc.)¹⁹ – and since the *Sphaerica* appears in the well-known list of Gerard's translations compiled by his students²⁰, we may safely attribute the translation to Gerard.

The Latin Manuscripts

P: Paris, BnF, lat. 9335, ff. 1r-19v, ca. 1200²¹.

R: Vatican, lat. 1548, ff. 25r-50v, 14c.²²

V: Vatican, Ottob. lat. 2234, ff. 54ra-64rb, 14c.²³

M: Madrid, Biblioteca Nacional, lat. 10010, ff. 1v-13r, 14c.²⁴

Kg: Cracow, Jagiellonian Univ. Library, 1924, pp. 223-257, 13-14c.²⁵

O: Oxford, Bodleian Library, Auct. F.5.28, ff. 29v-51v, 13c.²⁶

Fi: Florence, Bibl. Nazionale Centrale, c.s. J.I.32, ff. 135v-165v, 13c.

Z: Venice, Bibl. Nazionale Marciana, 1647 (f.a. 332), ff. 261r-289r, 13c.

¹⁸Lorch [1996], 169–171.

¹⁹For Gerard's style, see Kunitzsch [1974], 104–110 and 214–217.

²⁰See the recent edition by Burnett, 276: Liber Theodosii de speris tractatus .III.

 $^{^{21}160}$ ff. See Björnbo. He here dates the codex to the 14th century, but Bernhard Bischoff dated it to the late 12th century (private communication, 1.9.89). The items are all or mostly translations by Gerard of Cremona.

 $^{^{22}}$ 76ff. See Nogara, 59–60.

 $^{^{23}}$ See Daly and Ermatinger, 22-23.

²⁴Formerly Toledo 98–24; 86 ff. See Millás Vallicrosa, 208–211.

²⁵318 pp. See Wisłocki, 461.

 $^{^{26}}$ For a full description of ${\bf O},$ ${\bf Fi},$ ${\bf Z}$ and ${\bf B},$ see Busard and Folkerts, 64–67, 49–51, 78–80, 36–38, resp.

B: Berlin, Staatsbibliothek, lat. qu. 510, ff. 94v-112, 13c.
Ps: Paris, BnF, lat. 7399, ff. 139v-173v, 14c.²⁷
Va: Vatican, Reg. lat. 1069, ff. 1r-44r²⁸.

The above manuscripts have been collated for the edition. The following manuscripts have not been included in the collation:

Db: Dresden, Sächsische Landesbibliothek, Db 86, ff. 128r–158v, 13c.²⁹
Pt: Paris, BnF, lat. 3359, ff. 89r–114v, mid-14c.³⁰
Od: Oxford, Bodleian Library, Digby 178, ff. 107r–111r, 15c.³¹
Bf: Berlin, Staatsbibliothek, lat. fol. 633, ff. 1r–47v, 15c.³²
G: Glasgow, University Library, Hunt. 394, pp. 1–83, late 15c.³³
Mq: Milan, Biblioteca Ambrosiana, Q 69 sup., ff. 41r–68v³⁴.

The following "mixed" manuscripts, which carry the Campanus text until III 10 and then go to the end with the Gerard translation, have also not been used in the collation:

Cracow, Jagiellonian Univ. Library, 1924, pp. 207–222, 13–14c. Schweinfurt, Stadtbibliothek, H 81 (not paginated; 2nd part), 16c. Vatican, lat. 3380, ff. 1r–24r, 16c.

The Edition

Arabic

The Arabic text was established by comparing \mathbf{A} with \mathbf{N} : in cases of disagreement the reading was decided by reference to the Greek, in the new edition by Claire Czinczenheim. The entire readings of \mathbf{A} and \mathbf{N} are to be found either in the text or in the apparatus. \mathbf{H} , containing as it does numerous faults of orthography and other trivial mistakes, is not regularly recorded, but only in cases of doubt in the \mathbf{AN} text. In general, trivial grammatical mistakes in pointing and in such matters as the orthography of the *hamza* have been silently corrected. As for the readings reported in the apparatus, a quotation from one manuscript

 27 See Cat. Paris, 351-352.

 $^{^{28}\}mathrm{This}$ is the only item in the codex. There is no printed description.

²⁹Described in Busard and Folkerts, 41–43. Not used because of physical damage. The manuscript has readings similar to **O** in the *Sphaerica* and also in Robert of Chester's version of Euclid (see *ibid.*, 43).

³⁰173 ff. See *Cat. BN*, 279–287.

³¹115 ff. See Macray, 190–192.

³²There is no printed description of this codex. It contains only Theodosius' Sphaerica and De habitationibus.

³³158 ff. See Young and Aitkin, 314–315.

³⁴70 ff. See Gabriel, 324–325.

is reproduced exactly (or as exactly as possible) as it appears in the manuscript, but when text from two or more manuscripts is quoted, correct pointing and orthography are imposed on it.

Angle brackets, < >, are used to include material added by the editors and not directly supported by the manuscripts. Square brackets, [], enclose readings that are uncertain because of physical damage.

Latin

The Latin text was established by transcribing \mathbf{P} , which appears to be, of all the Latin manuscripts, the closest to the Arabic; the transcription was then compared with ten other manuscripts, taken in approximate order of agreement with the Arabic (see the order of the list of manuscripts, above). In every case the reading was decided by comparison with the Arabic and with Gerard's known style. The titles of the three books and the colophons are edited only from \mathbf{P} .

The equivalences of the diagram letters may be seen in Table 1. In the edition of the Latin we have kept to Gerard's lettering, with the exception of his *thel*, 7, \vdash and *Ge*, which we render by Υ , \mathring{Z} , \mathring{D} and \mathring{G} , respectively.

General Remarks

The numbering of the propositions is not constant in the various texts. Throughout, the Latin agrees with \mathbf{A} , and this is the numbering we have adopted for the edition. Table 2 is a conspectus of the proposition numbers in Books I and II. In Book III the numbering is constant – though in \mathbf{H} III 7 is omitted in its place and added at the end of the codex.

The mathematical summary, printed after the text, is a translation of the mathematical argument of the Arabic text. It also contains, in the footnotes, remarks about the presence or absence of some elements of meaning in the Arabic and Latin; but no attempt has been made to make such remarks complete.

It is our intention to provide future scholars with reliable information rather than to draw conclusions ourselves. But we note that right at the start there is extra material in the Arabic-Latin texts not to be found in the Greek edited by Czinczenheim: definitions 6 and 7, on the distances of circles from the centre of the sphere, and much of definitions 8–11, on the inclination of planes. Further obvious differences appear in the early theorems of Book I.