1. ASTRONOMY IN THE *REPUBLIC*

1.1. INTRODUCTION

Arithmetic is one of the five branches of mathematics which the future philosopher-rulers of the city outlined in the *Republic* will study for a decade before they move on to dialectic, i.e. philosophy, according to book 7, 537b7–c3. It is introduced in book 6 together with another branch, geometry, in the context of the simile of the divided line. Socrates is presented as asking Glaucon, his codiscussant and Plato's brother, to imagine a line divided into two unequal parts, liken one to sensibles and the other to intelligibles and then divide each part in the same proportion. The first section of the "sensible" part contains shadows, images and reflections on all kinds of surfaces; the second contains the objects that cast shadows and are pictured or reflected (509d6–510b1). Arithmetic and geometry are introduced in the description of the contents of the "intelligible" part of the divided line (510b2–511c2):

> Σκόπει δὴ αὖ καὶ τὴν τοῦ νοητοῦ τομὴν ᾗ τμητέον.
>
> Πῇ;
>
> Ἧι τὸ μὲν αὐτοῦ τοῖς τότε μιμηθεῖσιν ὡς εἰκόσιν χρωμένη ψυχὴ ζητεῖν ἀναγκάζεται ἐξ ὑποθέσεων, οὐκ ἐπ' ἀρχὴν πορευομένη ἀλλ' ἐπὶ τελευτήν, τὸ δ' αὖ ἕτερον [τὸ] ἐπ' ἀρχὴν ἀνυπόθετον ἐξ ὑποθέσεως ἰοῦσα καὶ ἄνευ τῶν περὶ ἐκεῖνο εἰκόνων, αὐτοῖς εἴδεσι δι' αὐτῶν τὴν μέθοδον ποιουμένη.
>
> Ταῦτ', ἔφη, ἃ λέγεις, οὐχ ἱκανῶς ἔμαθον, ἀλλ' αὖθις <**
>
> **> ἦν δ' ἐγώ· ῥᾷον γὰρ τούτων προειρημένων μαθήσῃ. οἶμαι γάρ σε εἰδέναι ὅτι οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς καὶ τὰ τοιαῦτα πραγματευόμενοι, ὑποθέμενοι τό τε περιττὸν καὶ τὸ ἄρτιον καὶ τὰ σχήματα καὶ γωνιῶν τριττὰ εἴδη καὶ ἄλλα τούτων ἀδελφὰ καθ' ἑκάστην μέθοδον, ταῦτα μὲν ὡς εἰδότες, ποιησάμενοι ὑποθέσεις αὐτά, οὐδένα λόγον οὔτε αὑτοῖς οὔτε ἄλλοις ἔτι ἀξιοῦσι περὶ αὐτῶν διδόναι ὡς παντὶ φανερῶν, ἐκ τούτων δ' ἀρχόμενοι τὰ λοιπὰ ἤδη διεξιόντες τελευτῶσιν ὁμολογουμένως ἐπὶ τοῦτο οὗ ἂν ἐπὶ σκέψιν ὁρμήσωσι.
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> Πάνυ μὲν οὖν, ἔφη, τοῦτό γε οἶδα.
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> Οὐκοῦν καὶ ὅτι τοῖς ὁρωμένοις εἴδεσι προσχρῶνται καὶ τοὺς λόγους περὶ αὐτῶν ποιοῦνται, οὐ περὶ τούτων διανοούμενοι, ἀλλ' ἐκείνων πέρι οἷς ταῦτα ἔοικε, τοῦ τετραγώνου αὐτοῦ ἕνεκα τοὺς λόγους ποιούμενοι καὶ διαμέτρου αὐτῆς, ἀλλ' οὐ ταύτης ἣν γράφουσιν, καὶ τἆλλα οὕτως, αὐτὰ μὲν ταῦτα ἃ πλάττουσίν τε καὶ γράφουσιν, ὧν καὶ σκιαὶ καὶ ἐν ὕδασιν εἰκόνες εἰσίν, τούτοις μὲν ὡς εἰκόσιν αὖ χρώμενοι, ζητοῦντες δὲ αὐτὰ ἐκεῖνα ἰδεῖν ἃ οὐκ ἂν ἄλλως ἴδοι τις ἢ τῇ διανοίᾳ.
>
> Ἀληθῆ, ἔφη, λέγεις.
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> Τοῦτο τοίνυν νοητὸν μὲν τὸ εἶδος ἔλεγον, ὑποθέσεσι δ' ἀναγκαζομένην ψυχὴν χρῆσθαι περὶ τὴν ζήτησιν αὐτοῦ, οὐκ ἐπ' ἀρχὴν ἰοῦσαν, ὡς οὐ δυναμένην τῶν ὑποθέσεων ἀνωτέρω ἐκβαίνειν, εἰκόσι δὲ χρωμένην αὐτοῖς τοῖς ὑπὸ τῶν κάτω ἀπεικασθεῖσιν καὶ ἐκείνοις πρὸς ἐκεῖνα ὡς ἐναργέσι δεδοξασμένοις τε καὶ τετιμημένοις.
>
> Μανθάνω, ἔφη, ὅτι τὸ ὑπὸ ταῖς γεωμετρίαις τε καὶ ταῖς ταύτης ἀδελφαῖς τέχναις λέγεις.
>
> Τὸ τοίνυν ἕτερον μάνθανε τμῆμα τοῦ νοητοῦ λέγοντά με τοῦτο οὗ αὐτὸς ὁ λόγος ἅπτεται τῇ τοῦ διαλέγεσθαι δυνάμει, τὰς ὑποθέσεις ποιούμενος οὐκ ἀρχὰς ἀλλὰ τῷ ὄντι ὑποθέσεις, οἷον ἐπιβάσεις τε καὶ ὁρμάς, ἵνα μέχρι τοῦ ἀνυποθέτου ἐπὶ τὴν τοῦ παντὸς ἀρχὴν ἰών, ἁψάμενος αὐτῆς, πάλιν αὖ ἐχόμενος τῶν ἐκείνης ἐχομένων, οὕτως ἐπὶ

τελευτὴν καταβαίνῃ, αἰσθητῷ παντάπασιν οὐδενὶ προσχρώμενος, ἀλλ’ εἴδεσιν αὐτοῖς δι’ αὐτῶν εἰς αὐτά, καὶ τελευτᾷ εἰς εἴδη.

> “Look now at how the intelligible part must be divided.”
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> “How?”
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> “In this manner: the soul is forced to study one part of it from hypotheses, using things that were imitated earlier on as images, not ascending to a starting point but descending to an endpoint, but with regard to the other part, it ascends from a hypothesis to an unhypothetical starting point and approaches it without its images, with and through the forms themselves.”
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> “I did not get what you just said,” he replied. “But again <**.”
>
> “**>,” I said. “You will understand my point more easily after the following. As you know, I am sure, the students of geometry, arithmetic and the like lay down odd and even, figures, three kinds of angle and other things akin to these in each field, and as if they knew these things, turning them into hypotheses, they do not deign to give either to themselves or to others an account of what is hypothesized, assuming that it is clear to everybody, but start from their hypotheses and go through the subsequent stages to arrive consistently at what they set out to investigate.”
>
> “I certainly know this,” he said.
>
> “So you also know that they use visible shapes and argue about them, but actually do not think about them but about those things that the visible shapes resemble, their proofs concerning the square itself and the diagonal itself, not that diagonal they draw, and so on–that is, they use as images the shapes they make up and draw, of which there are also shadows and reflections in water, in their attempt to see those things themselves that one can see with no other means than thought.”
>
> “It is true,” he said.
>
> “These were the intelligibles I was talking about in whose study the soul is forced to rely on hypotheses without ascending to a starting point, since it cannot transcend its hypotheses, but using visible images that are considered to be clearer than the originals and thus prized.”
>
> “I see,” he said, “that you are talking about geometry and its kindred fields.”
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> “So you can see that the other section of the intelligible part I was talking about is what reason itself grasps with the power of dialectic, employing hypotheses not as starting-points but as genuine hypotheses, let us say as footholds and launchers, so as to reach what is unhypothetical, the principle of all, and then, having gotten hold of it, turn back and, grasping what depends on it, descend in this manner to the end-point, using no sensibles whatsoever but the forms themselves through themselves to themselves, and end up with forms.”

The first section of the “intelligible” part of the line contains the objects studied in mathematics via their visible images and problematic definitions, “hypotheses”;[1] the second contains the forms studied in philosophy without such aids.

Plato seems to view what is studied in mathematics as forms approached in a particular way. Below in *R.* 7, 533a10–c6, he has Socrates say that mathematics sees beings in a dream via unclear hypotheses for which not accounts are given, not in the state of wakefulness, as dialectic does. Here he has Socrates give the square *itself* with the diagonal *itself* as example of an object studied in geometry. Forms have been introduced as the only beings at the end of *R.* 5, in the description of the philosophers (473e5–480a13) after the claim that, unless philosophers rule or rulers philosophize, humankind’s troubles will not end (473c11–e4). Philosophers want to learn about forms such as the beautiful *itself*, the intelligible and unchanging

1 For hypotheses in the divided-line simile as definitions see Bostock (2009) 13.

objects of knowledge, each of which is unique but, since it is associated with the changeable sensibles, appears everywhere as many, e.g. beautiful things: the latter resemble their form but are subject to change, and thus cannot possibly be objects of knowledge but only of opinion, though according to non-philosophers they are the only existents.[2] Forms are not sensible because that they are immaterial.[3] They seem to be conceived as eternal or atemporal entities not existing in space.[4] In terms of the traditional ontological categories, they are usually thought to be abstract properties, not definable in observational terms.[5] As mathematical objects, forms are best regarded as abstract particulars since in mathematics what does not look like a thing, e.g. a function, is regularly treated as such.[6]

If mathematical objects are forms, the sections of the "intelligible" part of the divided line do not answer to two different kinds of intelligibles, one studied by mathematics, the other by philosophy, each discipline approaching its objects in its own way, but to the distinct ways in which philosophy and mathematics approach intelligibles of a single type, forms; if so, the sections of the "sensible" part of the divided line similarly do not correspond to two kinds of sensibles but to two distinct ways in which sensibles are approached, and forms can be objects of belief and sensibles of knowledge insofar as they are related to forms.[7]

We can restore to mathematics its own objects, intelligible ones distinct from forms but similar to them in two crucial respects that explain the use of the same terminology for the description of both kinds of entities, if we rely on the testimony of Aristotle. According to Aristotle, between forms and sensibles Plato wedged mathematical objects as a third kind of existents. These are similar to forms in two respects, hence intelligible, and to sensibles in another: the so-called intermediates are similar to forms, and differ from sensibles, in that they are eternal and cannot move or suffer any change, but also resemble sensibles, and differ from forms, in that for each of them there are many alike (*Metaph*. A 6, 987b14–18).

Just as there is a single form of beauty over the many beautiful sensible things, there is a single form over the many intermediates that are alike. Aristotle contrasts mathematical numbers, each of which contains its predecessor plus one unit, from those numbers that do not each contain their predecessors: mathematical numbers consist of undifferentiated and combinable units, but each number of the other type has its own units, not combinable with those of any other number (*Metaph*. M 6, 1080a12–35). The units in the numbers of either type lack magnitude, are partless and indivisible (cf. *Metaph*. M 6, 1080b16–20, and 8, 1083b8–17). Aristotle calls numbers which are sets of undifferentiated and indivisible units "monadic" (from μονάς, "unit"). Numbers with combinable units are intermediates since Aristotle

2 The discussion of the Good at *R*. 6 contrasts the oneness of an intelligible form with the many sensibles associated with, or "participating" in, it and thus also named after it (507b1–9); for the contrast see also *Phd*. 78c10–79a5.

3 For their immateriality see *Sph*. 246a7–c3.

4 See *Ti*. 48e2–52d1 and the description of beauty itself in *Smp*. 210e2–211b5. On whether forms are timeless or eternal see Sorabji (1983) 108–112.

5 See e.g. Fine (1999) 215 n. 1.

6 See Gowers (2008) 10. For a précis of Platonism in mathematics see Brown (2005) 59–60.

7 See Fine (1999). All forms can thus be only those of mathematical objects; see ch. 2.9.

says that for each one of them there exist infinitely many alike (*Metaph*. M 7, 1081a5–12); numbers consisting of non-combinable units, on the other hand, are said to be forms since a form is unique (*Metaph*. M 7, 1082b24–28). There is no hint in Plato's works that he introduced the distinction between intermediates and forms, just as nothing in *R*. 6, 510b2–511c2, hints that his example of an object studied in mathematics, the square itself with the diagonal itself, is not a form but an intermediate.[8] Not unreasonably, scholars have doubted that Plato had put forth this distinction even in his discussions with members of the Academy.[9]

It is implausible that Aristotle simply foisted it on him, however. An Academic argument for the existence of forms discussed in his *On forms* was based on the objects of the sciences: the objects of a science exist; they are not particulars, for these are infinitely many and undetermined but each object of a science is single and determined; thus there are things that are different from particulars, and these things are forms (Alex. Aphr. *in Metaph*. 79.8–11 Hayduck). Aristotle would agree with Plato that the objects of mathematics are determined in the sense that each of them is what it is since e.g. lines are just lines, without breadth and depth, and straight ones lack any curvature (Euc. *El*. 1 Def. 2 and 4). But he might object that each one of them is not unique: no number of lines etc. is assumed in geometry, so if the argument shows that there must exist some things different from sensible particulars in that they are determined, these things will not be forms, each of which is unique, but form-like in that each of them must be eternal and not subject to change or motion if it is not a sensible particular and what is not a sensible particular is eternal and does not change or move. Assuming that there are other eternal things that do not change or move, the forms, each of which is unique, Aristotle could argue that Plato is committed to intermediates, thereby trying to answer a question raised by the passage from the *Republic* translated above. In it Plato talks about the visible shapes used in geometrical proofs and about which the geometers seem to argue, such as a square drawn with its diagonal, one of a great many such shapes that can be or are drawn or exist in the physical world, and he also distinguishes them from the intelligible objects that are truly studied in geometry, such as the square itself with the diagonal itself. These are described by him in the same way that he describes forms: the square itself with the diagonal itself seems to answer to the intelligible form of beauty, the beautiful itself, a single being that is associated with many sensibles and appears everywhere as square things, such as the figures drawn in the context of geometrical proofs,

8 E.g. Yang (1999) argues that it is an intermediate, Franklin (2012) 494–497 that it is a form.

9 For references see Arsen (2012) 201, who argues in favor of mathematical intermediates. For a survey of older literature against intermediates in Plato's ontology see Brentlinger (1963). He attempts to strike a middle position suggesting that as intermediates, in a weaker sense than that in which the term is employed by Aristotle, Plato must have regarded the objects of the definitions of arithmetic and geometry: definitions are said in *Ep*. 7, 342a7–344d2, to be one of the four means by which everything is knowable, so their objects, which are different from both sensibles and forms, whose representations they are, are indispensible to mathematical knowledge, actually of forms, a crucial fact mathematicians fail to grasp, ending up treating erroneously as objects of mathematical knowledge what are only means to it. Brentlinger does not explain, however, why Aristotle speaks of Plato's intermediates as eternal, like forms.

which are like it but subject to change. Now, how do the things defined in the hypotheses of mathematics fit into the apparently exhaustive division of existents into sensibles and their forms, which sensibles resemble? As defined in Euclid's *Elements*, which can be reasonably assumed to reflect the state of mathematics in Plato's time, the square of our example is not a sensible object (*El.* 1 Def. 19 and 22); nor can it be the square itself, for the latter is unique but geometry does not limit the number of objects defined as square. As will be argued next, Plato probably viewed the things defined in the hypotheses of mathematics as neither eternal nor lacking change and motion but as rarefied mental images of sensibles in the simplest cases (e.g. visible squares). Since these sensibles share in a form (e.g. the square itself) and the mental images of them cannot but participate in the relevant form themselves, these images and their (mentally) visualized motions and changes can be used to approach the form. But Aristotle could insist polemically that Plato's own principles force him to introduce intermediates as a new category of existents and go on to charge him with ontological profligacy.

That the objects studied in geometry such as lines and circles are not like any sensible objects was pointed out by Protagoras. Aristotle's testimony, embedded in his critique of Plato's supposed theory of intermediates, presupposes the immobility of mathematical objects as conceived by Plato (*Metaph.* B 2, 997b12–998a6):

> ἔτι δὲ εἴ τις παρὰ τὰ εἴδη καὶ τὰ αἰσθητὰ τὰ μεταξὺ θήσεται, πολλὰς ἀπορίας ἕξει· δῆλον γὰρ ὡς ὁμοίως γραμμαί τε παρά τ' αὐτὰς καὶ τὰς αἰσθητὰς ἔσονται καὶ ἕκαστον τῶν ἄλλων γενῶν· ὥστ' ἐπείπερ ἡ ἀστρολογία μία τούτων ἐστίν, ἔσται τις καὶ οὐρανὸς παρὰ τὸν αἰσθητὸν οὐρανὸν καὶ ἥλιός τε καὶ σελήνη καὶ τἆλλα ὁμοίως τὰ κατὰ τὸν οὐρανόν. καίτοι πῶς δεῖ πιστεῦσαι τούτοις; οὐδὲ γὰρ ἀκίνητον εὔλογον εἶναι, κινούμενον δὲ καὶ παντελῶς ἀδύνατον· ὁμοίως δὲ καὶ περὶ ὧν ἡ ὀπτικὴ πραγματεύεται καὶ ἡ ἐν τοῖς μαθήμασιν ἁρμονική· καὶ γὰρ ταῦτα ἀδύνατον εἶναι παρὰ τὰ αἰσθητὰ διὰ τὰς αὐτὰς αἰτίας· εἰ γὰρ ἔστιν αἰσθητὰ μεταξὺ καὶ αἰσθήσεις, δῆλον ὅτι καὶ ζῷα ἔσονται μεταξὺ αὐτῶν τε καὶ τῶν φθαρτῶν. ἀπορήσειε δ' ἄν τις καὶ περὶ ποῖα τῶν ὄντων δεῖ ζητεῖν ταύτας τὰς ἐπιστήμας. εἰ γὰρ τούτῳ διοίσει τῆς γεωδαισίας ἡ γεωμετρία μόνον, ὅτι ἡ μὲν τούτων ἐστὶν ὧν αἰσθανόμεθα ἡ δ' οὐκ αἰσθητῶν, δῆλον ὅτι καὶ παρ' ἰατρικὴν ἔσται τις ἐπιστήμη καὶ παρ' ἑκάστην τῶν ἄλλων μεταξὺ αὐτῆς τε ἰατρικῆς καὶ τῆσδε τῆς ἰατρικῆς· καίτοι πῶς τοῦτο δυνατόν;…ἀλλὰ μὴν οὐδὲ τῶν αἰσθητῶν ἂν εἴη μεγεθῶν οὐδὲ περὶ τὸν οὐρανὸν ἡ ἀστρολογία τόνδε. οὔτε γὰρ αἱ αἰσθηταὶ γραμμαὶ τοιαῦταί εἰσιν οἵας λέγει ὁ γεωμέτρης (οὐθὲν γὰρ εὐθὺ τῶν αἰσθητῶν οὕτως οὐδὲ στρογγύλον· ἅπτεται γὰρ τοῦ κανόνος οὐ κατὰ στιγμὴν ὁ κύκλος ἀλλ' ὥσπερ Πρωταγόρας ἔλεγεν ἐλέγχων τοὺς γεωμέτρας), οὔθ' αἱ κινήσεις καὶ ἕλικες τοῦ οὐρανοῦ ὅμοιαι περὶ ὧν ἡ ἀστρολογία ποιεῖται τοὺς λόγους, οὔτε τὰ σημεῖα τοῖς ἄστροις τὴν αὐτὴν ἔχει φύσιν.[10]

> Again, if one posits intermediates besides forms and sensibles, he will face many difficulties. For it is clear that there will be lines besides the lines themselves and sensible lines, and similarly with the objects studied in each of the other sciences. Thus, since astronomy is one of them, there will be a cosmos besides the sensible cosmos and a Sun and a Moon and all the other celestial objects. How can one believe these things, however? It is not plausible that the cosmos is immobile and that it moves is completely impossible. The same, moreover, applies to the objects studied in optics and mathematical harmonics. These, too, cannot exist apart from sensibles for the same reasons. For, if there are intermediate sensibles and sensations, it

10 *Metaph.* B 2, 997b32–998a4 = Protag. DK 80 B 7.

> is clear that there will also be animals between the animals themselves and the perishable animals. A further difficulty is what beings must be studied in these sciences. For, if geodesy differs from geometry only in this respect, i.e. in that the former is concerned with sensibles and the latter with non-sensibles, then it is clear that besides medicine there will be another science between the medicine itself and the medicine of sensibles, and similarly with each of the other sciences. But how is this possible?...On the other hand, astronomy would be neither about sensible lines nor about this cosmos. For there are no sensible lines such as those a geometer's proofs are concerned with (since nothing sensible is as straight or circular, given that a ruler touches a circle not at a point but as Protagoras used to say taking exception to the geometers), nor are the motions and helixes of the cosmos like those about which astronomy gives proofs, nor do celestial objects have the same nature as points.

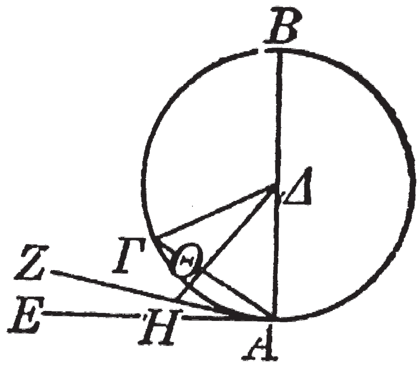

Fig. 1

Protagoras pointed out the obvious clash between what geometry shows, i.e. that a straight line touches a circle at a point, in Euclid's *Elements* the porism of theorem 3.16, and what we can see right away when we use a ruler to draw a straight line touching a circle drawn with a compass, i.e. that the ruler, and the line we draw with it, touches the circle over a region of perceptible magnitude. The theorem, whence it follows that a straight line touches a circle at a point, asserts that (a) a straight line drawn at right angles to a circle's diameter falls outside the circle, (b) between this straight line and the circumference no other straight line can be interposed and (c) the angle of the semicircle, the mixed angle of the circle's diameter and its circumference, is greater, and the similarly mixed angle of the circumference and the straight line at right angles to the diameter less, than any acute rectilinear angle.[11] In fig. 1 we assume a straight line ΓA at right angles to a circle's diameter falling within the circle AΓB and also draw ΓΔ, but since ΓΔ = ΔA as radii of a circle and the angle ΔAΓ is right, the angle ΔΓA of triangle ΔAΓ is also right, hence a triangle's two internal angles are absurdly both right, and it is thus clear that a straight line perpendicular to the diameter BA at A cannot share with the circle another point: this line must be AE, which is seen, though, to share with the circle not a point but a stretch of its circumference, as Protagoras pointed out. (b) is a further example of this clash between geometry and sensibles: it

11 On the angle of the circumference and its tangent see Heath (1956) vol. 2, 39–43.

asserts that AZ does not exist, though it can be easily drawn. For, if ΔH is drawn from Δ perpendicular to AZ, then since the angle AHΔ will be right, the angle ΔAH will be less than a right angle and thus the straight line ΔA will be greater than the straight line ΔH, in the triangle ΔAH the greater side subtending the greater angle (Euc. *El.* 1.19): but ΔA is a radius of the circle and cannot be greater than ΔH which has been drawn from Δ perpendicular to AZ outside the circle, unless the part ΔΘ of ΔH, Θ being the point where ΔH cuts the circumference, is absurdly greater than the whole. If AZ does not exist, no acute angle BAZ exists and (c) follows immediately: the mixed angle formed by the diameter BA and the circumference AΓ is greater, and the mixed angle formed by the straight line AE at right angles to the diameter BA and the circumference AΓ less, than any acute rectilinear angle.

We do not know what lesson Protagoras drew from the fact that in geometry a straight line is shown to touch a circle at a point, whereas a line drawn with a ruler is seen to touch a circle drawn with a compass over a region of some length. But it is implausible that in this self-evident fact Plato could have seen evidence that the intelligible and mind-independent objects geometry really studies are unlimitedly many, eternal, non-sensible and "perfect" lines and circles, like those defined in the hypotheses of contemporary geometry: lines and circles that not only lack even the slightest deviations from true straightness and circularity but also cannot move and change, unlike all sensibles that resemble them and via them unique forms, i.e. the circle itself and the line itself, a single circle and a single line somehow different from, though as "perfect" and motionless as, the countlessly many intermediate or mathematical circles and lines that are like them and over which they each preside.

In the geometry of Plato's time basic geometrical objects were most probably defined through their kinematically visualized production, just as in the Euclidean *Elements*, where the sphere is generated by a rotating semicircle (*El.* 11, Def. 14):

> Σφαῖρά ἐστιν, ὅταν ἡμικυκλίου μενούσης τῆς διαμέτρου περιενεχθὲν τὸ ἡμικύκλιον εἰς τὸ αὐτὸ πάλιν ἀποκατασταθῇ, ὅθεν ἤρξατο φέρεσθαι, τὸ περιληφθὲν σχῆμα.
>
> A sphere is the figure comprehended when, the diameter remaining fixed, the semicircle is carried around and is restored to the same position from which it began to be moved.

The sphere is one of the three solid figures defined by Euclid in a kinematically visualized manner. The cone, too, is defined as the figure generated when a right triangle turns about one of the sides of its right angle (*El.* 11, Def. 18):

> Κῶνός ἐστιν, ὅταν ὀρθογωνίου τριγώνου μενούσης μιᾶς πλευρᾶς τῶν περὶ τὴν ὀρθὴν γωνίαν περιενεχθὲν τὸ τρίγωνον εἰς τὸ αὐτὸ πάλιν ἀποκατασταθῇ, ὅθεν ἤρξατο φέρεσθαι, τὸ περιληφθὲν σχῆμα.
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> A cone is the figure comprehended when, one of the sides around the right angle of a right-angled triangle remaining fixed, the triangle is carried around and is restored to the same position from which it began to be moved.

Similar also is the Euclidean definition of the cylinder (*El.* 11, Def. 21):

> Κύλινδρός ἐστιν, ὅταν ὀρθογωνίου παραλληλογράμμου μενούσης μιᾶς πλευρᾶς τῶν περὶ τὴν ὀρθὴν γωνίαν περιενεχθὲν τὸ παραλληλόγραμμον εἰς τὸ αὐτὸ πάλιν ἀποκατασταθῇ, ὅθεν ἤρξατο φέρεσθαι, τὸ περιληφθὲν σχῆμα.
>
> A cylinder is the figure comprehended when, one of the sides about the right angle of a right-angled parallelogram remaining fixed, the parallelogram is carried around and is restored to the same position from which it began to be moved.

Would Plato have thought that the kinematic definitions of the cone etc. describe each a class of infinitely many geometrical objects that exist independently of our mind and are actually unsusceptible to motion and change, hence eternal, or rarefied mental images of sensibles participating in forms, like the sensibles themselves, and also capable of undergoing mentally visualized motions and changes? The first alternative is unlikely given Plato's view on squaring, addition, and application of areas (*R*. 7, 527a1–b11): the geometers' talk of performing these operations on the objects of their study presupposes that these objects do come to be and also pass away, though they are in fact eternal beings, which suggests that it cannot be they themselves that are treated by the geometers as if they were not eternal but some other, non-eternal things similar to, and thus participating in, them. These things can only be sensibles and mentally formed images thereof.[12] That an intelligible object, be it a supposed intermediate or a form, instantiates perfectly a hypothesis of mathematics is ruled out by the non-spatiality of beings, which means that they do not have shape (see *Phdr*. 247c4–d1), and has been plausibly denied by Proclus: he points out that in the realm of the immaterial causes extended things exist without extension, divided things without division, magnitudes without magnitude, figures without shape (*in Euc*. 54.1–8 Friedlein). The forms of geometrical objects cannot be geometrical, nor can form-numbers be monadic, which is how numbers would be defined in the hypotheses of Plato's contemporary arithmetic (*El*. 7, Def. 1–2).[13]

Examples of hypotheses are given in *R*. 6, 510b2–511c2, from geometry and arithmetic, and are followed by a cursory reference to their kin in each of the other branches of mathematics. These are astronomy and harmonics if the distinction in the following book between plane and solid geometry is not presupposed here. If it is, then solid angles and solid figures will be defined in the hypotheses of solid geometry; this must hold as much for the immature solid geometry in the dramatic time of the *Republic* as for its advanced descendant that will be of propedeutical use to philosophy (*R*. 7, 528a10–c7). Plato, moreover, has Socrates regard not only solid geometry but also astronomy and harmonics as not sufficiently developed to be propedeutically useful to dialectic. This service will again be rendered by an advanced astronomy and harmonics of the future (*R*. 7, 528e1–531c8).

12 Cf. above n. 9 on Brentlinger's suggestion. Franklin (2012) argues in a similar manner that Plato viewed intermediates as "theoretical fictions". It should be noted that Plato does not despise visualization in mathematics. In this he is remarkably modern; see Mancosu (2005) 13 –30.

13 Wedberg (1955) 66, 80–84 and 120 denies that form-numbers can be monadic since forms are simple; he also cites van der Wielen (1941) ch. 7, esp. 87–89, according to whom Aristotle misrepresented Plato's form-numbers (van der Wielen [1941] is extensively reviewed in Cherniss [1947] 235–251). See also Findlay (1974) 56–57, Tarán (1981) 13–29 and (1991) 206–224; cf. below 1.5.2.